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Re-Entry Wake in an Earth-Fixed Coordinate System

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Introduction

INVESTIGATIONS of the re-entry wake have largely been based on a solution of the steady-state problem [1–3] It is true that, for many problems of interest in aerodynamics, phenomenological effects follow so rapidly on a change of freestream conditions that the steady results, used on a quasi-steady basis, offer realistic answers In general, this is not true for the wake problem, since wake cooling is a relatively slow process in many cases, so that it may be necessary to consider the actual trajectory of a re-entry vehicle The present note is a description of such a technique, summarizing the results of Ref 4 The simplifying assumptions are such that the present results are contained implicitly in the steady-state solutions under the coordinate transformation $x = tV_\infty$, with x the distance along body trajectory, t the time, and V_∞ the freestream velocity In Ref 5, the steady-state results are used to describe the wake in the same manner as the present description Even so, neither the present results nor the results of Ref 5 can be considered quasi-steady, since the complete instantaneous wake solution is not merely a function of the current freestream conditions, but depends strongly on the history of vehicle motion along its trajectory

In the present case, the solution of the complete flow field is carried out by considering the body flow field and near wake separately from the far wake (following the techniques of the cited steady-solution references)

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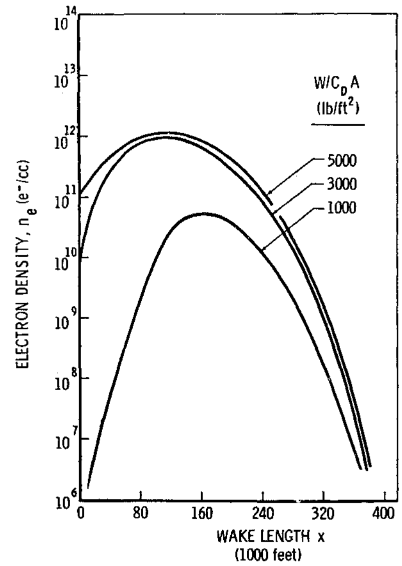

Fig 1 Axial electron density distribution — hemisphere at 20,000-ft alt

Body Flow Field and Near Wake

This portion of the flow field is considered to be an isentropic expansion from the conditions behind the shock Streamlines are thus constant entropy lines The equation

$$\left(\frac{dp}{dT}\right)_S = \frac{(1/T)(\partial h/\partial T)_p}{(RZ/p) - (1/T)(\partial h/\partial p)_T} \tag{1}$$

is integrated with respect to temperature until the pressure decays to the freestream value, using the thermodynamic properties of real air in chemical equilibrium [6] In Eq (1), T is the temperature, p the pressure, S the entropy, h the enthalpy, Z the compressibility, and R is the gas constant In order to determine the conditions behind the shock from which to start the integration, a bow shock shape is assumed For highly blunted bodies, such as hemispheres, the Van Hise[7] shock correlation is used A modified correlation[8] is used to give more accurate shock shapes in those cases for which the Van Hise correlation is not strictly applicable, that is, for slightly blunted bodies such as high fineness ratio spherically capped cones The radial distribution of flow properties at the beginning of the far wake is obtained from continuity considerations

The Far Wake

As the nose cone proceeds along its trajectory, it leaves behind a radial distribution of flow properties at each axial station in the wake This distribution, which is a function of altitude, velocity, and the body properties, as was ex-

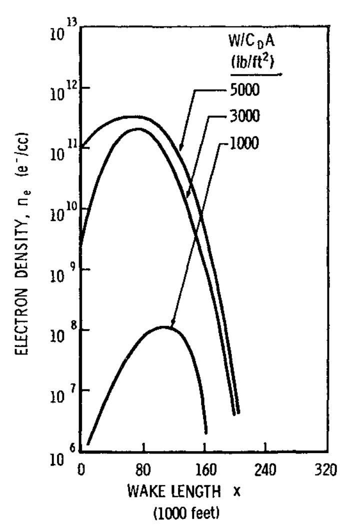

Fig 2 Axial electron density distribution — 15° blunted cone at 20,000-ft alt

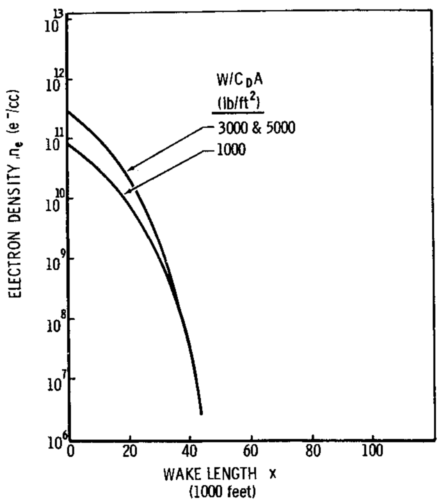

Fig 3 Axial electron density distribution—15° blunted cone at 100,000-ft alt

plained in the preceding section, is taken as the initial character of the far wake In the present simplified calculations, only enthalpy distributions are considered The velocity of the far wake is neglected so that it can be treated as a cooling stationary column of air with large radial enthalpy gradients The axial enthalpy gradients will be much smaller than the radial, so that the latter will have the predominant effect over the short times being considered Cooling in the wake is then governed by the equation

$$\rho \frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\mu}{Pr}\frac{\partial h}{\partial r}\right) \tag{2}$$

where ρ is the density, t the time, r the radial cylindrical coordinate, and μ the viscosity

An integral method is used to solve this equation Making use of the assumed profile

$$h/h_\infty = 1 + a(t)\{1 + [R_1{}^2/b(t)]\}^{-(1+\epsilon)} \tag{3}$$

where a and b are functions of time, ϵ is a constant, and

$$R_1{}^2 = \int_0^{r^2} \frac{\rho}{\rho_{ef}}\, d(r^2) \tag{4}$$

we can integrate (2) from $r = 0$ to $r = \infty$ to obtain

$$a(t)b(t) = a(0)b(0) \tag{5}$$

Now, taking the limit of (2) as $r \to 0$ yields†

$$\frac{da}{dt} = -\frac{4(1+\epsilon)\mu a^2}{\rho_{ref} Pr\, a(0)b(0)} \tag{6}$$

Equation (6) is integrated numerically The result is a variation of enthalpy with time at each point along the vehicle trajectory When the body is at a particular altitude of interest, the distribution of enthalpy at each point in the wake is determined from the duration of time since the body has passed that point These times are obtained from the trajectory that is being followed

The exponent $1 + \epsilon$ is a constant depending on the initial enthalpy profiles For all cases that have been run, it has been found that Eq (3) offers a good approximation to the initial conditions There are no discontinuities across the zero strength shock in the far wake (i e , the bow shock has become a Mach line), so that it is allowable to use (3) over the whole range $0 \leqslant r \leqslant \infty$

† This equation is similar to that obtained in Ref 1 under the transformation $tV_\infty = x$

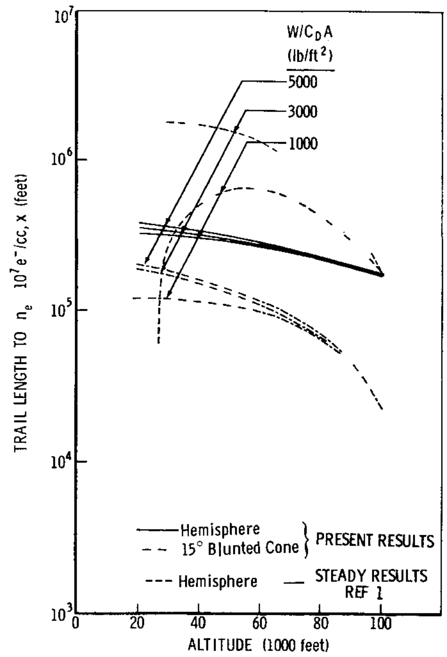

Fig 4 Altitude and ballistic coefficient effect on wake length

The viscosity coefficient in Eq (6) is obtained from Sutherland's formula, and an average value of $Pr = 0\ 71$ has been assumed Since the flow is considered to be in equilibrium, electron density is a function of the pressure and enthalpy

Discussion

Calculations have been carried out for two bodies with a 1-ft base radius, a hemisphere, and a spherically capped cone of 15° half angle with a ratio of nose radius to base radius of $\frac{1}{6}$ The axial distribution of electron density in the wake was determined for each body at altitudes of 20,000, 60,000, and 100,000 ft, for W/C_DA of 1000, 3000, and 5000 psf W/C_DA is the ballistic coefficient, W the weight, C_D the drag coefficient, and A the reference area The entry velocity and angle are 23,400 fps and −20°, respectively Typical results are shown in Figs 1–3 Figures 1 and 2 show that it is possible for transparent windows to exist in the wake for certain frequency radar The trail length for these radar will be only the distance included under the curves

Wake length is presented as a function of altitude in Fig 4, with ballistic coefficient as a parameter The anomalies of the steady calculations are readily apparent at low altitudes However, it would seem that the steady calculations can be used above 100,000-ft alt because of the relatively rapid cooling and the nearly constant vehicle velocity, at least for the cases considered here

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Effect of Variable Lewis Number on Heat Transfer in a Binary Gas

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Introduction

SEVERAL authors have implicitly considered the effect of variable Lewis number in a binary diffusion process, particularly for the case of local equilibrium in a stagnation-point boundary layer [1–4] Anderson[4] found that the stagnation-point heat-transfer rate was predicted within 5% when the average Lewis number in the boundary layer was used in the Fay and Riddell formula In this note, the effect of variable Lewis number is explicitly determined for the equilibrium and frozen Couette flow of an ideal dissociating diatomic gas The results[5] indicate that the constant Lewis number giving the same heat-transfer rate as the variable Lewis number solution may be simply expressed in terms of the freestream and wall values of Lewis number and is only weakly dependent on freestream velocity and wall temperature

Analysis

The equations of state, enthalpy, and equilibrium atom mass fraction for an ideal dissociating diatomic gas are (in the notation of Clarke[6])

$$p = (1 + Ca)\rho R_0 T/W_m \tag{1}$$

$$H = (4 + Ca)R_0 T/W_m + CaD \tag{2}$$

$$Ca = [1 + (pW_m/\rho_d R_0 T)\exp(W_m D/R_0 T)]^{-1/2} \tag{3}$$

The gas transport properties are based on Moore,[7] who has shown that, to a good approximation, the ratio of mixture viscosity to molecular viscosity and the reciprocal Schmidt number $1/Sc = Le/Pr$ of a binary mixture vary linearly with atom mole fraction In terms of atom mass fraction, the viscosity and Lewis number become

$$\mu = C_2 T^{0\ 75}(1 + 1\ 234Ca)/(1 + Ca) \tag{4}$$

$$Le = (C_3 Pr)(1 - 0\ 234Ca)/(1 + Ca) \tag{5}$$

$Pr = 0\ 74$ and $C_3 Pr = 1\ 45$ were used in the analysis, since the Prandtl number variations are quite small compared with those of Lewis number

Integration of the momentum and energy equations for a Couette flow leads to the following results:

Shear

$$\tau = \mu(du/dy) = \text{const} \tag{6a}$$

Velocity Profile

$$\frac{y}{\delta} = \frac{2}{Re_\delta C_f}\int_0^{u/u_\delta}\frac{\mu}{\mu_\delta}\,d\left(\frac{u}{u_\delta}\right) \text{ where } Re_\delta = \frac{\rho_\delta u_\delta \delta}{\mu_\delta} \tag{6b}$$

Enthalpy Profile

$$H - H_w + D[L(Ca) - L(Ca_w)] + (Pr/2)u^2 = -q(Pr/\tau)u \tag{6c}$$

where $L(Ca) = (Le - 1)Ca$ for constant Le and $= 1\ 789 \ln(1 + Ca) - 1\ 339Ca$ for variable Le q is the heat flux rate, D is the dissociation energy per unit mass of atoms, and the subscripts w and δ refer to the boundary values at the surface $y = 0$, where $u = 0$, and at the outer edge of the parallel viscous flow, where $y = \delta$, $u = u_\delta$ From Eqs (6) we obtain the following values for the skin-friction and heat-transfer coefficients:

$$C_f = \frac{2\tau}{\rho_\delta u_\delta^2} = \frac{2}{Re_\delta}\int_0^1 \frac{\mu}{\mu_\delta}\,d\left(\frac{u}{u_\delta}\right) \tag{7}$$

$$St = -q/\rho_\delta u_\delta(H \ - H_w) = (C_f/2Pr)[1 + D\{L(Ca_\delta) - L(Ca\ )\}/(H \ - H_w)] \tag{8}$$

The recovery enthalpy H is the value of H_w when $q = 0$ Using Eqs (6), we obtain the enthalpy recovery factor:

$$\theta = \frac{H_r - H_\delta}{H_{t\,t\,1} - H_\delta} = Pr + \frac{2D}{u_\delta^2}[L(Ca_\delta) - L(Ca\ )] \tag{9}$$

The solution is now complete except for the determination of the atom concentration in the layer For equilibrium flow, $Ca = Ca$ [Eq (3)] For frozen flow, the continuity of species equation may be written as

$$\frac{d^2Ca}{du^2} - \frac{1}{Sc}\frac{dSc}{dCa}\left(\frac{dCa}{du}\right)^2 = 0 \tag{10}$$

which, with Eq (5), is integrated to give the atom concentration in the layer as

Variable Le

$$\frac{u}{u_\delta} = \frac{1\ 234\ln[(1 + Ca)/(1 + Ca_w)] - 0\ 234(Ca - Ca_w)}{1\ 234\ln[(1 + Ca_\delta)/(1 + Ca_w)] - 0\ 234(Ca_\delta - Ca_w)} \tag{11a}$$

Constant Le

$$Ca = Ca_w + (Ca_\delta - Ca_w)u/u_\delta \tag{11b}$$

The freestream atom concentration Ca_δ was arbitrarily assumed to correspond to equilibrium The wall atom concentration Ca_w is determined by the equation $(\rho DamdCa/dy)_w = \Gamma S$, where S is the atom mass flux rate per unit area to the wall, and Γ is the fraction of these atoms which recombine Assuming that the atoms near the wall have a Maxwellian velocity distribution, we find

$$S = \left(\frac{Ca_w}{1 + Ca_w}\right)\frac{p}{(\pi R_0 T_w/W_m)^{1/2}}$$

Using these results in Eqs (11) then gives the following ex pressions for the wall atom concentration:

Variable Le

$$1\ 234\ln\left(\frac{1 + Ca_\delta}{1 + Ca_w}\right) - 0\ 234(Ca_\delta - Ca_w) = \frac{0\ 5103u_\delta}{\tau_w}\left(\frac{Ca_w}{1 + Ca_w}\right)\frac{\Gamma p}{(\pi R_0 T_w/W_m)^{1/2}} \tag{12a}$$

Constant Le

$$Ca_w = \tfrac{1}{2}[b + (b^2 + 4Ca_\delta)^{1/2}]$$

where

$$b = Ca_\delta - 1\ \frac{- Sc_w \Gamma u_\delta p}{\tau_w(\pi R_0 T_w/Wm)^{1/2}} \tag{12b}$$

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